The Design and Implementation of An Abstract Interpreter for OCaml Programs

A Preliminary Report on the Salto Analyser

Benoît Montagu, Inria

Innia

ML workshop, Seattle — 2023, September 8th

The Salto Project

- **What:** static analysis for OCaml programs https://salto.gitlabpages.inria.fr/
- ▶ Where: Inria Rennes
- Who:



P. Lermusiaux



T. Genet



T. Jensen



B. Montagu





► Funding: *Ínría* + M Nomadic Labs

I SEE UNCAUGHT EXCEPTIONS imaflip.com

Short-term goals:

- Detect uncaught exceptions
 - ► User-provided assertions
 - Missing exception handlers (e.g., Division_by_zero)
 - Out of bounds accesses for arrays, strings, ...
 - Polymorphic comparison on functions
- Detect illegal uses of unsafe functions (e.g., String.unsafe_get)

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- Detect unhandled algebraic effects
- Detect some undefined behaviours (e.g., sensitivity to evaluation order)

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- Support for the Obj module

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This talk

Static Analyses for Uncaught Exceptions

Two families of static analyses:

- ▶ Type and effect systems:
 - Modular, good performance
 - Limited precision for user-provided assertions

Xavier Leroy and François Pessaux. "Type-Based Analysis of Uncaught Exceptions". In: ACM Trans. Program. Lang. Syst. 22.2 (2000), pp. 340–377. DOI: 10.1145/349214.349230

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- Extensions of control-flow analyses (CFA):
 - Not modular, more costly
 - Decent precision for user-provided assertions

Kwangkeun Yi. "Compile-time Detection of Uncaught Exceptions in Standard ML Programs". In: Static Analysis, First International Static Analysis Symposium, SAS'94, Namur, Belgium, September 28-30, 1994, Proceedings. Ed. by Baudouin Le Charlier. Vol. 864. Lecture Notes in Computer Science. Springer, 1994, pp. 238–254. DOI: 10.1007/3-540-58485-4_44

▶ Principle:

For every reachable sub-expression *e* of a program, compute:

- A superset of the values that *e* may evaluate to, and
- ► A superset of the exceptions *e* might raise
- ▶ An approximation of the call stack where the exception was raised

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- Novelty: an abstract domain to represent recursively defined sets of values
- ▶ Implementation: uses a dynamic fixpoint solver

What We Have Achieved So Far

- ► An abstract interpreter (big-step style) that supports:
 - Higher-order programs
 - Mutually-recursive functions
 - ➔ Algebraic values, deep pattern matching
 - Integers, strings, characters...
 - Exceptions
 - Modules and functors (first class, non-recursive)
 - No mutable state yet
 - No laziness
 - No objects/classes
 - No OCaml 5 features
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- Demo! list_filter map_merge mc91 insert_sorted_list

A finite representation for *recursively defined* sets of *untyped* values:

$$v^{\sharp} \in \mathbb{V}^{\sharp} = \{ \text{ ints } = d \in \mathbb{Z}^{\sharp}; \\ \text{variants } = \{c_1 \mapsto v^{\sharp}; \dots; c_n \mapsto v^{\sharp}\}; \\ \text{pairs } = (v^{\sharp}, v^{\sharp}); \\ \text{funs } = \{(\lambda^{\ell} \mathbf{x}.t) \mapsto [\mathbf{x}_1 \mapsto v^{\sharp}; \dots; \mathbf{x}_n \mapsto v^{\sharp}]; \dots \} \} \\ \mid \mathsf{T}$$

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 The widening operator detects some regularity and introduces the µs

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Example: A set of continuations (for CPSed factorial)

$$\mu \alpha. \left\{ \mathsf{funs} = \left\{ \begin{array}{l} (\lambda^{\ell_1} x. x) \mapsto []; \\ (\lambda^{\ell_2} x. \, \mathsf{k} \, (x * n)) \mapsto [\mathsf{n} \mapsto \{\mathsf{ints} = [1, +\infty]\}; \mathsf{k} \mapsto \alpha]; \end{array} \right\} \right\}$$

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Abstract Domain: Important Remarks

The design of the abstract domain draws inspiration from:

- Equi-recursive types + union types
- ► Type Graphs (analysis of Prolog programs)

Pascal Van Hentenryck, Agostino Cortesi and Baudouin Le Charlier. "Type Analysis of Prolog Using Type Graphs". In: *The Journal of Logic Programming* 22.3 (Mar. 1995), pp. 179–209. DOI: 10.1016/0743–1066(94)00021–w

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These abstract values admit two representations:

As graphs

C Efficient algorithms for union, intersection, inclusion, emptiness test, widening, minimisation, ...

► As terms, with bound variables

🖒 Permits hash-consing/memoisation

c This is crucial to obtain decent performance (~10× improvement!)

Related Work (1/2)

Pessaux & Leroy's effect type system:

- ▶ They infer recursive types, using unification
- ▶ They support arrow types, row variables for effects: enables modular analysis
- They do not infer abstract closures: Incurs a loss of information when using functions as first-class values
- Limited support for sets of integers: Int[1:Pre; 3:Pre] Int[Τ] Int[ρ]
 We support any abstract domain for integers (non-relational so far)

Control-Flow Analyses:

- ▶ They always avoid recursion in the abstract domain
- Recursion is obtained by means of indirections through an abstract heap

$$\begin{cases} \mathsf{funs} = \left\{ \begin{array}{l} (\lambda^{\ell_1} \mathbf{x}. \mathbf{x}) \mapsto []; \\ (\lambda^{\ell_2} \mathbf{x}. \mathbf{k} (\mathbf{x} \ast \mathbf{n})) \mapsto [\mathbf{n} \mapsto p_{\mathbf{n}}; \mathbf{k} \mapsto p_{\mathbf{k}}]; \end{array} \right\} \end{cases}$$

where: $\hat{h}(p_{\mathbf{n}}) = \{\mathsf{ints} = [1, +\infty]\}$
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- Mimics the behaviour of a compiler: Values are allocated in the heap
- ▶ In practice: inhibits sharing of equivalent abstract values
- There is a finite number of abstract pointer names: names are chosen based on a (finite) abstraction of the call stack
- The abstract heap is global:

This prevents refining information when some control-flow branch is taken

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Consider the following program: if x < 42 then e1 else e2

- To analyse e1 with precision, we need to exploit the fact that (x < 42) evaluated to true</p>
- This is done by running a **backward analysis** on the expression (x < 42)

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Forward analysis et backward analyses depend on each other!

- ► A problem in all interprocedural analyses
- Solution: use a **dynamic fixpoint solver**

Defining Static Analysers Using Dynamic Fixpoint Solvers

val fix: $((X.t \rightarrow Y.t) \rightarrow (X.t \rightarrow Y.t)) \rightarrow (X.t \rightarrow Y.t)$

Computes a post-fixpoint of the functional passed as argument

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m CP Allows to define a big-step analyser using open recursion:

```
fix @@ fun analyse (t, env) -> match t with
| Var x -> Env.get env x
| Lam (x, t) -> D.make_closure x t (Env.restrict env (fv (Lam (x, t))))
| App(t1, t2) ->
let v2 = analyse (t2, env) in
if D.is_bot v2 then D.bot else
let v1 = analyse (t1, env) in
D.joins (D.closures v1)
  (fun (x, t, env0) -> analyse (t, Env.add x v2 env0))
```

fix implements the iteration strategy of the analyser and tracks dynamic dependencies to avoid unnecessary recomputations

val fix: $((X.t \rightarrow Y.t) \rightarrow (X.t \rightarrow Y.t)) \rightarrow (X.t \rightarrow Y.t)$

Computes a post-fixpoint of the functional passed as argument

▶ Idea pioneered by work on Prolog analysis

Pascal Van Hentenryck, Agostino Cortesi and Baudouin Le Charlier. "Type Analysis of Prolog Using Type Graphs". In: *The Journal of Logic Programming* 22.3 (Mar. 1995), pp. 179–209. DOI: 10.1016/0743–1066(94)00021–w

► Later re-emphasized (in a simpler setting)

David Darais et al. "Abstracting definitional interpreters (functional pearl)". In: *Proc. ACM Program. Lang.* 1.ICFP (2017), 12:1–12:25. DOI: 10.1145/3110256

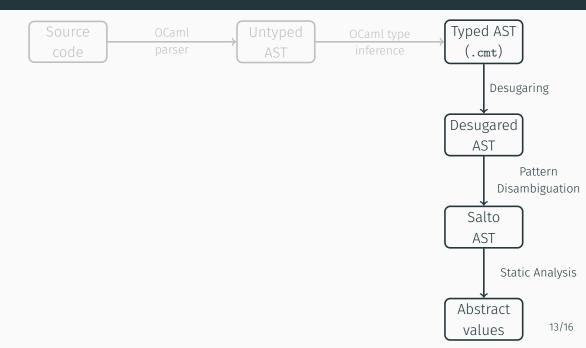
Actually used in a static analyser for C programs

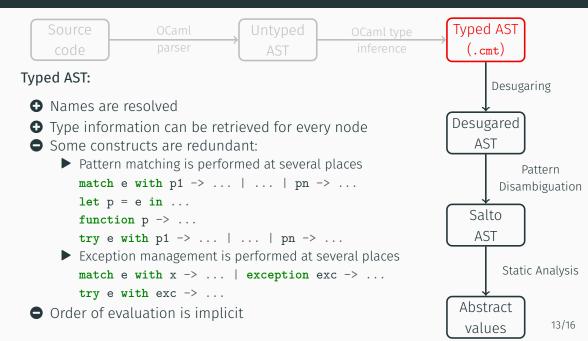
Vesal Vojdani et al. "Static race detection for device drivers: the Goblint approach". In: Proceedings of the 31st IEEE/ACM International Conference on Automated Software Engineering, ASE 2016. ACM, 2016, pp. 391–402. DOI: 10.1145/2970276.2970337

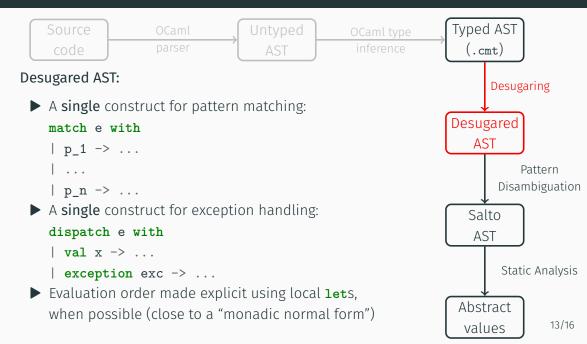
Defining Static Analysers Using Dynamic Fixpoint Solvers

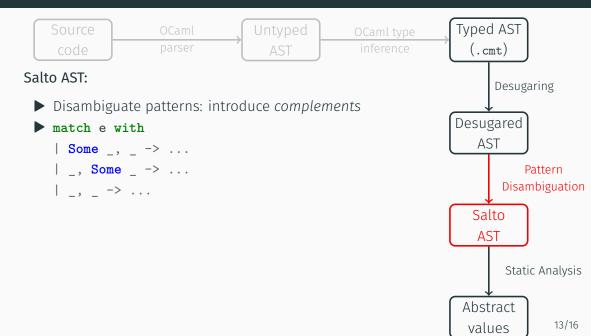
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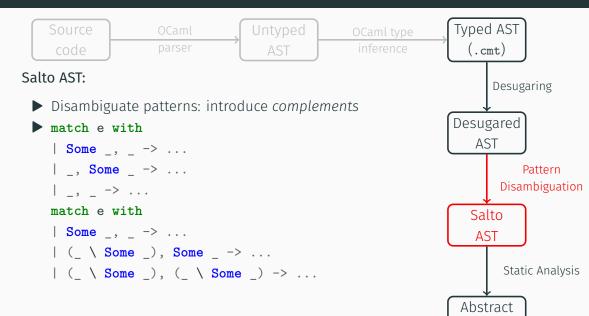
▶ You've heard about fixpoint solvers and static analysers at ICFP this week! Sven Keidel, Sebastian Erdweg and Tobias Hombücher. "Combinator-Based Fixpoint Algorithms for Big-Step Abstract Interpreters". In: *Proceedings of the ACM on Programming Languages* 7.ICFP (Aug. 2023), pp. 955–981. DOI: 10.1145/3607863





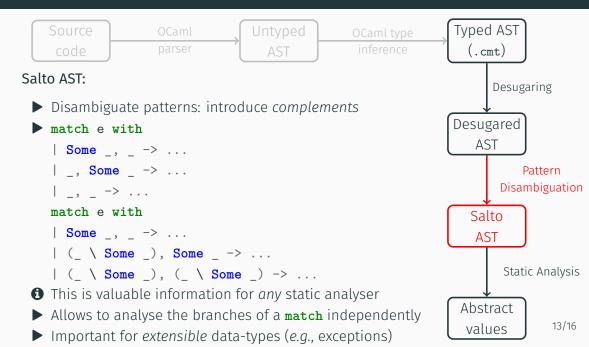


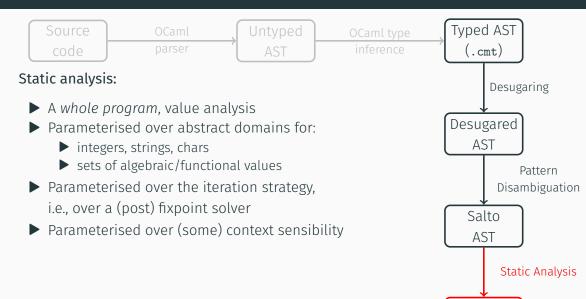




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values

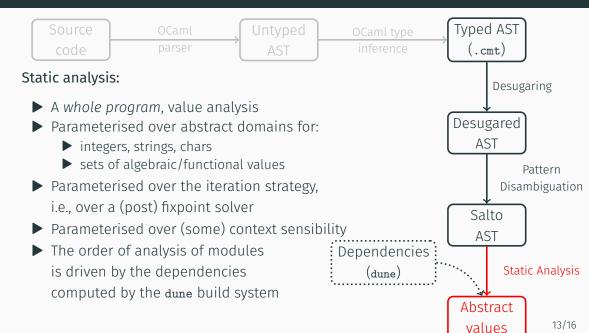




Abstract

values

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Code component	Code size
AST transformations	~ 3000 LoC
Abstract domain for values	$\sim 3500 \; {\rm LoC}$
Core of the abstract interpreter	$\sim 4300 \; {\rm LoC}$
Fixpoint engine	$\sim 500 \; {\rm LoC}$

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267 test programs (\leq 200 LoC), featuring:

- ► Higher-order, direct style programs
- Church encodings
- CPS programs
- Defunctionalised programs
- Monadic programs
- ▶ Non-regular types, GADTs

Analysis times range from 200 ms to 2 mn

Relational analysis (especially: input/output relations)

Benoît Montagu and Thomas P. Jensen. "Stable Relations and Abstract Interpretation of Higher-order Programs". In: *Proc. ACM Program. Lang.* 4.ICFP (2020), 119:1–119:30. DOI: 10.1145/3409001

- Expressive and efficient relational domains for sets of trees are still an open problem
- ▶ Low-level representation of data (Оъј module)
- Algebraic effects (one-shot continuations)
- Multicore
- ► Signals
- Scalability of the analysis

Conclusion

Salto: Static Analyses for Trustworthy OCaml 🛛 🔀 OCaml

- ► A work in progress!
- An abstract interpreter for OCaml programs that detects uncaught exceptions
- ▶ Features an abstract domain for inductively defined sets of values
- Implemented using a dynamic fixpoint solver

raise Questions

https://salto.gitlabpages.inria.fr/

B. Montagu + P. Lermusiaux + T. Genet + T. Jensen

The Road Ahead (1)

Support more features of OCaml:

- Support mutable state
 - 🖒 References and mutable data-types
 - 🖒 Arrays
 - \mathcal{C} External state provided by the OS (e.g., file descriptors)
- ▶ Detect arithmetic overflows/underflows
- Detect problematic cases of pattern matching on mutable data
- Cyclic values, e.g.: let rec 1 = 1 :: 1
- The lazy construct
- ▶ Objects, classes, recursive modules...

Refine the analysis:

- Incorporate a narrowing phase to the fixpoint solver
- Exploit the types inferred by the OCaml compiler (reduced product)
- Specific abstract domains for strings, bytes, sets, maps, hash-tables...

- Minimisation is important to reduce memory consumption
- And also helps avoid some unnecessary computations thanks to memoisation
- **Example:** Peano numbers admit several equivalent representations

 $\mu\alpha.\{\text{variants} = \{ 0 \mapsto \cdot; S \mapsto \alpha \} \}$

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- Our minimisation algorithm canonises these three abstract values into the first one

- Improve error reporting and UI (LSP server?)
- ▶ Incremental changes of code
- "Explainable Abstract Interpretation"
- Produce examples of "bad" inputs
- Requires a lot of testing, engineering, time, and love!

► Type-based analysis of exceptions

Xavier Leroy and François Pessaux. "Type-Based Analysis of Uncaught Exceptions". In: ACM Trans. Program. Lang. Syst. 22.2 (2000), pp. 340–377. DOI: 10.1145/349214.349230

Control-flow analysis

Olin Shivers. "The Semantics of Scheme Control-Flow Analysis". In: Proceedings of the 1991 ACM SIGPLAN Symposium on Partial Evaluation and Semantics-Based Program Manipulation. PEPM '91. New York, NY, USA: Association for Computing Machinery, 1991, pp. 190–198. ISBN: 0897914333. DOI: 10.1145/115865.115884

Control-flow analysis using widening

Benoît Montagu and Thomas P. Jensen. "Trace-Based Control-Flow Analysis". In: PLDI '21: 42nd ACM SIGPLAN International Conference on Programming Language Design and Implementation, Virtual Event, Canada, June 20-25, 2021. Ed. by Stephen N. Freund and Eran Yahav. ACM, 2021, pp. 482–496. DOI: 10.1145/3453483.3454057 Analysis of Prolog with type graphs

Pascal Van Hentenryck, Agostino Cortesi and Baudouin Le Charlier. "Type Analysis of Prolog Using Type Graphs". In: *The Journal of Logic Programming* 22.3 (Mar. 1995), pp. 179–209. DOI: 10.1016/0743–1066(94)00021–w

Analysis of logic programs with tree grammars

Patrick Cousot and Radhia Cousot. "Formal Language, Grammar and Setconstraint-based Program Analysis by Abstract Interpretation". In: Proceedings of the seventh international conference on Functional programming languages and computer architecture - FPCA '95. ACM Press, 1995. DOI: 10.1145/224164.224199

Graph-based representations for sets of trees

Laurent Mauborgne. "Representation of Sets of Trees for Abstract Interpretation". PhD thesis. École Polytechnique, Nov. 1999. URL: https://www.di.ens.fr/ ~mauborgn/publi/t.pdf A relational abstract domain for trees with numeric data
 Matthieu Journault, Antoine Miné and Abdelraouf Ouadjaout. "An Abstract Domain for Trees with Numeric Relations". In: Programming Languages and Systems - 28th European Symposium on Programming, ESOP 2019, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2019, Prague, Czech Republic, April 6-11, 2019, Proceedings. Ed. by Luís Caires. Vol. 11423. Lecture Notes in Computer Science. Springer, 2019, pp. 724–751. DOI: 10.1007/978-3-030-17184-1_26

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▶ Approach followed by the Goblint static analyser

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